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TRANSVERSE VIBRATIONS OF BELLOWS EXPANSION JOINTS. PART I: FLUID ADDED MASS

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This paper presents the results of an analysis of the fluid-added mass in bellows expansion joints during bending vibrations. The added mass is shown to consist of two parts, one due to transverse rigid-body motion and the other due to distortion of the convolutions during bending. The latter component, neglected in previous analyses, is shown to be important for relatively short bellows, as are commonly used for expansion joints, and to become increasingly important for higher vibration modes. The distortion component has been determined using finite element analysis, and the results are presented in a graphical form for a typical range of bellows geometries. The total added mass is given in a form suitable for hand calculations.

1. INTRODUCTION

Bellows expansion joints are designed to absorb axial, transverse, and angular displacments in piping systems. However, their flexibility also makes them susceptible to self-excited vibrations (Gerlach, 1969; Weaver & Ainsworth, 1989). The onset of self-excited vibrations at a bellows natural frequency is typically quite sudden and the resulting bending stresses may produce fatigue failures in a relatively short period of time. Weaver & Ainsworth (1989) showed that the flow excitation was associated with a constant Strouhal number of 0·45, based on the mean flow velocity through the bellows and convolution pitch as the characteristic length. Thus, prediction of the flow velocity for self-excitation depends on knowledge of the bellows natural frequencies.

Most studies have concentrated on axial vibrations and axisymmetric natural frequency prediction methods of varying degrees of complexity (Gerlach 1969; EJMA 1980; Jakubauskas & Weaver 1992). Much less work has been published on transverse vibrations (EJMA 1980; Ting Xin Li *et al.* 1986), and none of these studies have considered the effects of the relative motion of the sides of a convolution, called convolution distortion, on the associated fluid-added mass. In the simplest terms, the inertial effect of the fluid on a bellows in axial vibration can be considered to be the mass of the fluid contained between the convolutions moving as a rigid body, as shown in Figure 1(a). On the other hand, the entire mass of the fluid contained in the bellows is associated with transverse vibrations as shown in Figure 1(b). However, Jakubauskas & Weaver (1996) have shown that convolution distortion in axial vibrations of bellows can produce significant added mass effects, especially for short bellows or higher vibration modes. This is caused by the relatively high accelerations of the fluid being squeezed in and out from between the convolutions during convolution distortion, as illustrated in Figure 1(c). No such analysis has been published for transverse or non-axisymmetric vibrations of bellows expansion joints. Such vibrations may

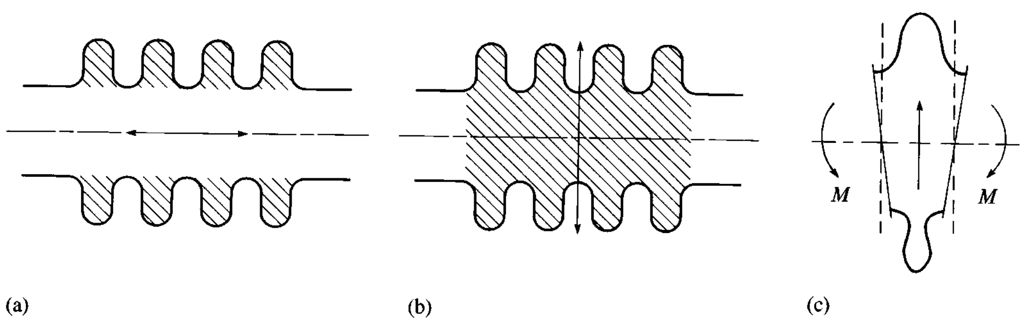

Figure 1. Fluid added mass in bellows: (a) axial vibration (assuming pure axial translation of rigid convolutions); (b) transverse vibration (assuming rigid body motion); (c) convolution distortion in bending.

be particularly important for double bellows or bellows immediately downstream of pipe elbows.

Part I of this study presents a theoretical model for the fluid-added mass of bellows expansion joints undergoing transverse vibrations. It would be straightforward but fairly cumbersome to compute the transverse natural frequencies of bellows, including the effects of added mass, by modelling the bellows as a shell containing fluid and using a finite element analysis. The particular approach used here was developed in an effort to make the prediction of natural frequencies amenable to hand calculations. The bellows is treated as a beam and analytical expressions are derived for the total fluid added mass, including the effects of convolution distortion. The pressure distribution on the convolution walls is determined using finite element analysis and the results are presented graphically in the form of an added mass coefficient. Calculations are carried out for a single bellows to demonstrate the relative importance of the various components of total bellows mass in the first four modes of transverse vibration. In Part II of this paper, these results are used in a theoretical model for transverse bellows vibration, and comparisons are made with both experiments and the predictions of EJMA (1980).

2. THEORETICAL DEVELOPMENT

In considering the transverse vibration of a bellows expansion joint, the total fluid added mass per unit length, m_f, is assumed to be comprised of two components: one associated with rigid-body motion, m_{f1}, and the second associated with convolution distortion, m_{f2}; thus,

$$m_f = m_{f1} + m_{f2}. \tag{1}$$

Only the rigid-body component, m_{f1}, is considered in the EJMA Standard (1980) and, as noted in the introduction, it is felt that neglect of the convolution distortion component could lead to significant error. Treating these components separately seems reasonable on physical grounds and makes the problem tractable.

Also treated separately is the fluid component of rotary inertia which is considered in Part II of this paper.

2.1. Rigid-Body Component, m_{f1}

This component is easily determined as the mass of fluid per unit length of bellows [see Figure 1(b)]. If R_m' is the radius of a cylinder containing the same volume of fluid per unit

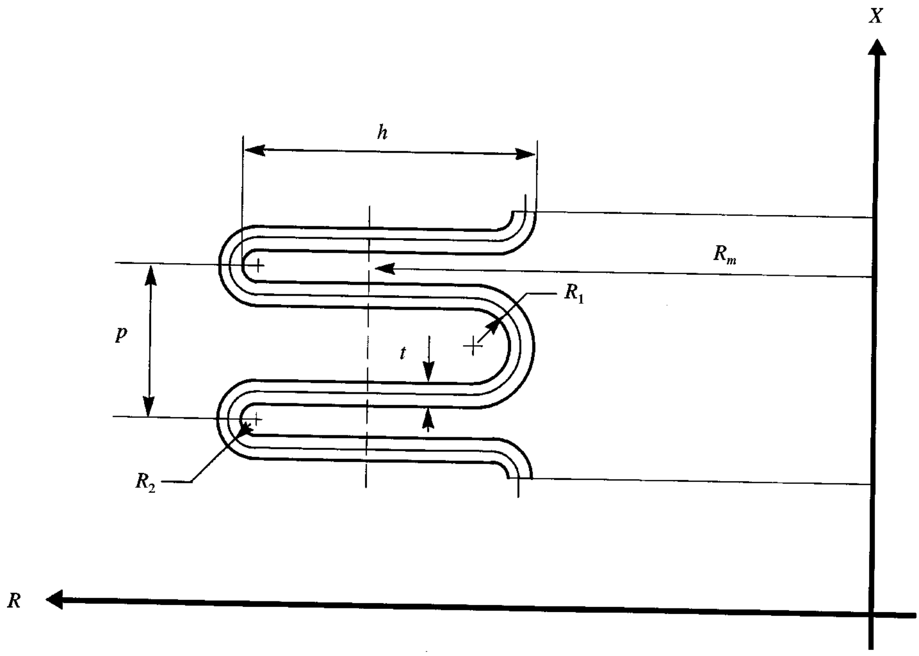

Figure 2. Bellows geometry.

length of bellows as the actual bellows, then

$$m_{f1} = \pi R_m'^2 \rho_f, \tag{2}$$

where ρ_f is the fluid density. Referring to Figure 2, it can be shown that

$$R_m' \simeq R_m - \frac{h}{2} + \frac{2hR_2}{p}, \tag{3}$$

where R_m is the mean radius of the bellows, h is the convolution height, R_2 is the meridional radius of the convolution crown, and p is the convolution pitch. Here, the half-thickness of the convolution has been ignored and, since h and R_2 are relatively small compared to R_m, the error in this approximation is considered to be negligible. Note that $p = 2(R_1 + R_2)$ and for the particular case $R_1 = R_2$, equation (3) reduces to $R_m' = R_m$. The rigid-body component of added mass per unit length can therefore be expressed as

$$m_{f1} = \pi \left(R_m - \frac{h}{2} + \frac{2hR_2}{p} \right)^2 \rho_f. \tag{4}$$

2.2. Convolution Distortion Component of Added Mass, m_{f2}

This component is much more complex than that for rigid-body motion as it involves the relative motion of the convolution surface, which varies along the length of the bellows and depends on both the bellows mode shape and the boundary conditions [see Figure 1(c)].

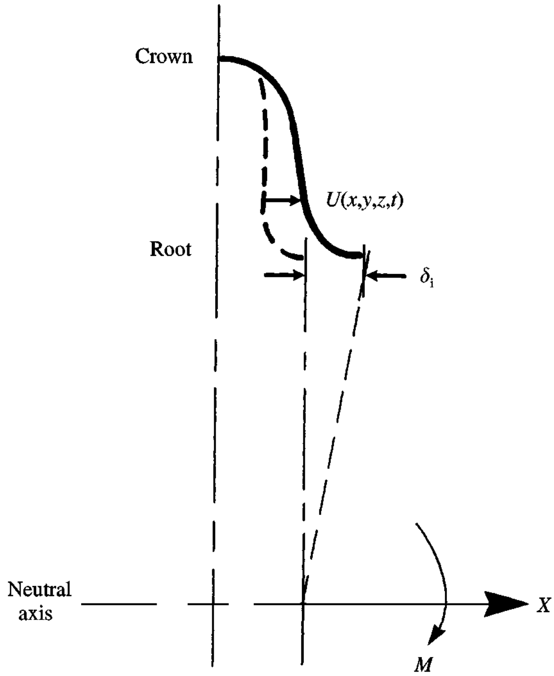

Figure 3. Initial and deformed shapes of bellows convolution due to moment M.

For simplicity, the bellows in transverse vibration will be assumed to behave like a beam in bending. While bellows are usually very short and may appear to be rather stubby beams, the peculiar geometry of the convolutions makes bellows very flexible in overall bending while maintaining substantial radial stiffness. Thus, significant rotations along the bellows axis can occur, with both shear and ovalling of the cross-section remaining negligible. This assertion is substantiated by the excellent agreement between theory and experiments (Jakubauskas 1996).

Consider one-half of the ith convolution at some point x along the bellows axis to have undergone a transverse displacement, $X_k(x)$, with associated moment, M. Since the added mass associated with transverse rigid-body motion was treated separately in the previous section, only the convolution distortion component caused by the moment M about the neutral axis is considered here. This shape distortion is defined by displacement $U(x, y, z, t)$ along the convolution as shown in Figure 3. For convenience, this displacement function will be referred to its maximum value which occurs at the convolution root, δ_i in Figure 3. Using the beam analogy, plane sections remain plane and the effective strain, ε, at the convolution root is given by

$$\varepsilon = \frac{2\delta_i}{p}, \tag{5}$$

where p is the convolution pitch. Noting that the distance of the convolution root from the neutral axis of bending is $R_m - h/2$ and using the geometrical relationship for curvature, the convolution root displacement can be written in terms of the beam displacement function in the kth mode, $X_k(x)$,

$$\delta_i = \frac{1}{2} p \left(R_m - \frac{h}{2} \right) X_k''(x), \tag{6}$$

where the prime denotes differentiation with respect to x. Defining the normalized to unity convolution surface displacement distribution as U^*, the relative displacement of the surface of the bellows convolutions along the entire bellows length may be written as

$$U = \delta_i U^* T(t) = \frac{1}{2} p \left(R_m - \frac{h}{2} \right) X_k''(x) U^* T(t), \tag{7}$$

where $T(t)$ gives the time dependence of the motion. If the fluid added mass for one half-convolution is defined as λ, then the kinetic energy for this mass in terms of the displacement along the half-convolution surface, S, can be written for the ith half-convolution as

$$K_i = \frac{1}{2} \iint_S \frac{\lambda \dot{U}^2}{S} \mathrm{d}S = \frac{\lambda \dot{T}^2(t)}{8} X_k''^2(x) \left(R_m - \frac{h}{2} \right)^2 p^2 \iint_S \frac{U^{*2}}{S} \mathrm{d}S. \tag{8}$$

Over a typical range of convolution geometries, $h/R_m = 0{\cdot}19 \pm 0{\cdot}04$ and $(R_2 - t/2)/R_m = 0{\cdot}0316 \pm 0{\cdot}0055$, calculations using the associated convolution displacement U^* show that the surface integral in equation (8) is nearly constant

$$\iint_S \frac{U^{*2}}{S} \mathrm{d}S \simeq 0{\cdot}132. \tag{9}$$

The total kinetic energy of the fluid due to convolution distortion is then the sum of the individual components along the bellows, i.e. for the $2N$ half-convolutions,

$$\begin{aligned} K &= 0{\cdot}132 \frac{\lambda \dot{T}^2(t)}{4} \left(R_m - \frac{h}{2} \right)^2 p \sum_i^{2N} X_k''^2(x) \frac{p}{2} \\ &\simeq 0{\cdot}033\, \lambda p \left(R_m - \frac{h}{2} \right)^2 \dot{T}^2(t) \int_0^l X_k''^2(x)\, \mathrm{d}x. \end{aligned} \tag{10}$$

Equation (10) gives the total kinetic energy of the fluid mass due to convolution distortion in terms of the added mass per one half-convolution, λ.

The fluid-added mass per unit length due to convolution distortion, m_{f2}, varies along the length of the bellows, depending on the local bending moment. However, if m_{f2} is considered as the average fluid added mass per unit length along the bellows for each mode shape, k, then the associated total kinetic energy is given by

$$K = \int_0^l \frac{m_{f2} \dot{w}_k^2}{2} \mathrm{d}x, \tag{11}$$

where w_k is the beam displacement in the kth mode,

$$w_k = X_k(x) T(t). \tag{12}$$

Substituting equation (12) into equation (11) yields

$$K = \frac{m_{f2}}{2} \dot{T}^2(t) \int_0^l X_k^2(x) \mathrm{d}x. \tag{13}$$

The average fluid added mass per unit length in the kth mode, m_{f2}, can now be determined in terms of the one half-convolution distortion added mass by equating equations (10) to (13).

$$m_{f2} = \alpha_{f2k}\lambda, \tag{14}$$

where

$$\alpha_{f2k} = 0{\cdot}066 \frac{\int_0^l \left(\frac{\mathrm{d}^2 X_k}{\mathrm{d}x^2}\right)^2 \mathrm{d}x}{\int_0^l X_k^2(x)\,\mathrm{d}x}\left(R_m - \frac{h}{2}\right)^2 p. \tag{15}$$

The integrals in equation (15) may be solved directly once the mode shape function, $X_k(x)$, is known. The determination of the fluid added mass per one half-convolution, λ, is covered in the following section.

2.3. Analysis for Half-Convolution Added Mass, λ

From classical hydrodynamics [for example, see Milne-Thomson (1968)], if the fluid is assumed to be inviscid and incompressible, the kinetic energy of the fluid for the ith convolution of the bellows can be written in terms of a velocity poetential, Φ, and the velocity normal to the bounding surface, $\partial\Phi/\partial n$, as

$$K_i = -\frac{\rho_f}{2}\iint_S \Phi \left.\frac{\partial \Phi}{\partial n}\right|_S \mathrm{d}S. \tag{16}$$

On the other hand, this kinetic energy can also be written in terms of the added mass per half-convolution, λ, and the convolution surface displacement, U, as given in equation (8):

$$K_i = \frac{\lambda}{2S}\iint_S \dot{U}^2\,\mathrm{d}S, \tag{17}$$

where S is the surface area of the bellows half-convolution. Equating kinetic energy expressions (16) and (17) gives the half-convolution added mass expression for the three-dimensional flow due to convolution distortion

$$\lambda = -\rho_f S \frac{\iint_S \Phi \frac{\partial \Phi}{\partial n}\,\mathrm{d}S}{\iint_S \dot{U}^2\,\mathrm{d}S\,.} \tag{18}$$

To use the above equation it is necessary first to calculate the velocity potential of the flow excited by the deformation of the convolution wall, U. Since this flow is three-dimensional, the velocity potential is described by the Laplacian [see, for example, Milne-Thomson (1968)],

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0. \tag{19}$$

For a bellows in bending it is readily seen that the x–z, and y–z planes in Figure 4 are planes of symmetry, while the x–y plane is one of anti-symmetry (for bending about the y-axis, the

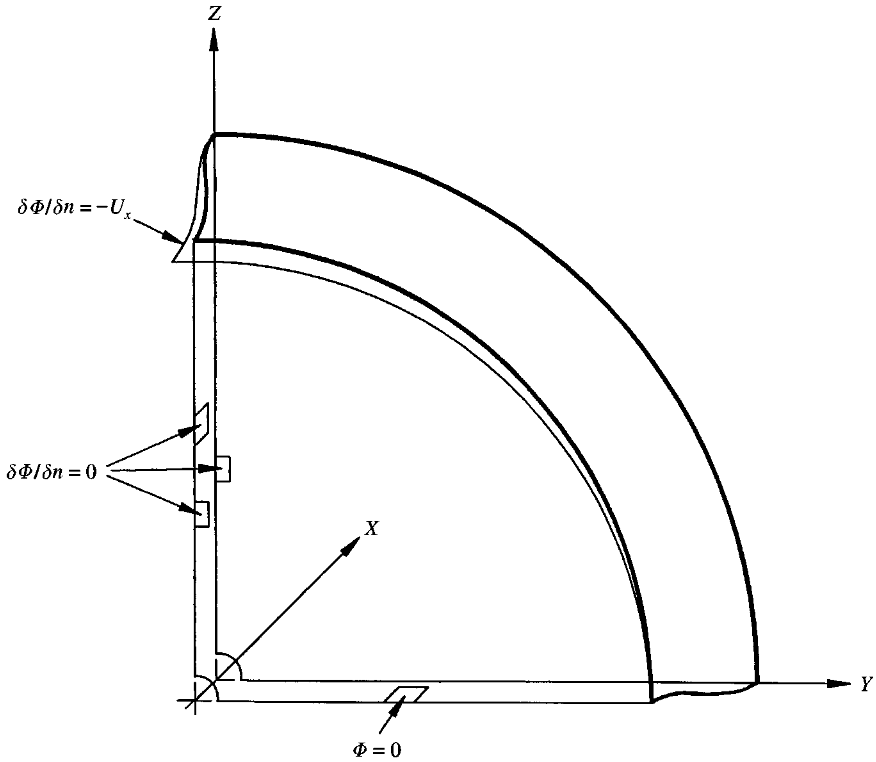

Figure 4. Fluid solution domain with boundary conditions.

convolution displacement in the negative z-direction is minus that in the positive z-direction). Therefore, it is sufficient to solve the problem for just one-eighth part of a convolution, with the appropriate boundary conditions.

Considering the convolution surface displacement, U, the components in the coordinate axes directions are defined as U_x, U_y, and U_z. It was shown by Jakubauskas (1991) in the case of axial deformation of the bellows, that the axial displacements (displacements along x, see Figure 4) are much larger than the radial ones. A similar relationship holds in the case of the bending deformation of the bellows as well when considering only the effects of convolution distortion, i.e., the radial and transverse displacements relative to the neutral axis of bending are negligible in comparison with the axial displacement. Therefore, in further considerations of the boundary condition on the convolution surface, just U_x will be taken into account, and U_z and U_y will be neglected. The surface displacement field on the convolution could be determined precisely using a finite element analysis of the bellows as a vibrating shell. However, a good estimate may be obtained much more simply by assuming that the dynamical displacemet field is the same as would be obtained under static loading and invoking the beam bending model. Thus, the bending deformation shape of the convolution wall in the x–z plane, U_x (at the top of the convolution in Figure 4) is determined from the analysis of the axial deformation shape of the bellows under static loading, $U_{x\mathrm{st}}$. This shape was determined for axial vibrations of bellows by Jakubauskas (1991) using an axisymmetric shell finite element analysis. Now, using the assumption that the bellows in transverse vibration behaves like a beam in bending, plane sections remain

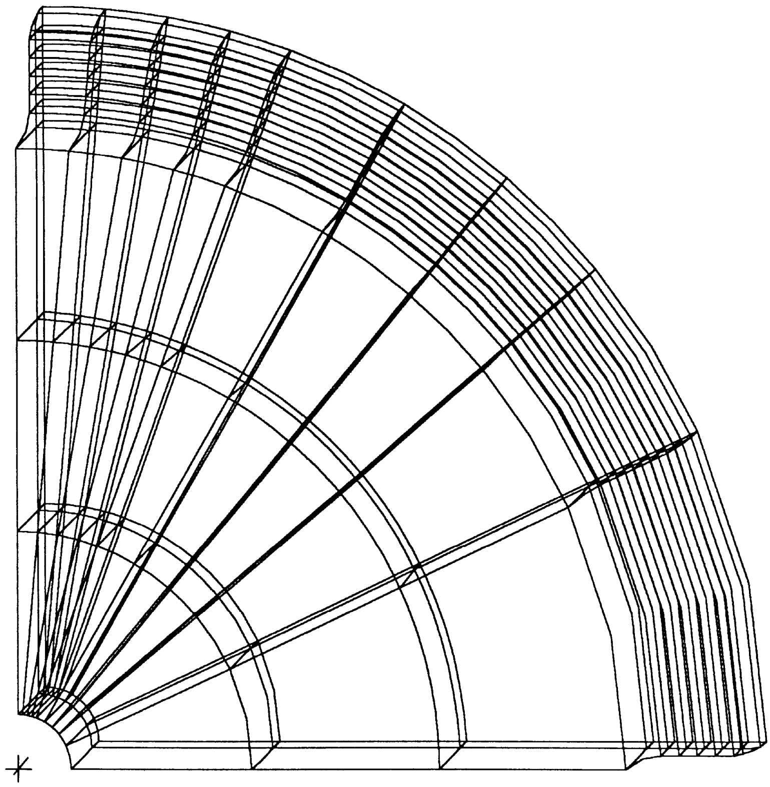

Figure 5. Fluid domain discretization.

plane and the radial convolution shape is a linear function of its distance, z, from the neutral plane of bending:

$$U \simeq U_x = U_{x\,\mathrm{st}} \frac{z}{R_m - \frac{1}{2}h} T(t). \tag{20}$$

The boundary condition for the fluid domain on the vibrating boundary is the impermeability condition

$$\left.\frac{\partial \Phi}{\partial n}\right|_S = -\,\dot{\mathbf{U}} \cdot \mathbf{n}. \tag{21}$$

Thus, the convolution shape, $U_{x\,\mathrm{st}}$, computed as noted above, was given the time-dependent bending distribution of equation (20) and used in equation (21) as the Neumann boundary condition on the bellows surface. The rest of the boundary conditions are shown in Figure 4. Since the added mass coefficient being calculated is an average value for the bellows, adjacent convolutions are assumed to have the same displacement, and no fluid will move between convolutions as the result of bending. This approximation leads to the homogeneous Neumann boundary condition on the plane between adjacent convolutions.

The velocity potential problem was solved using finite element analysis. The 20-noded isoparametric brick element was chosen to fit the curved domain boundary along the convolution wall. The domain was divided into 162 such elements, as shown in Figure 5.

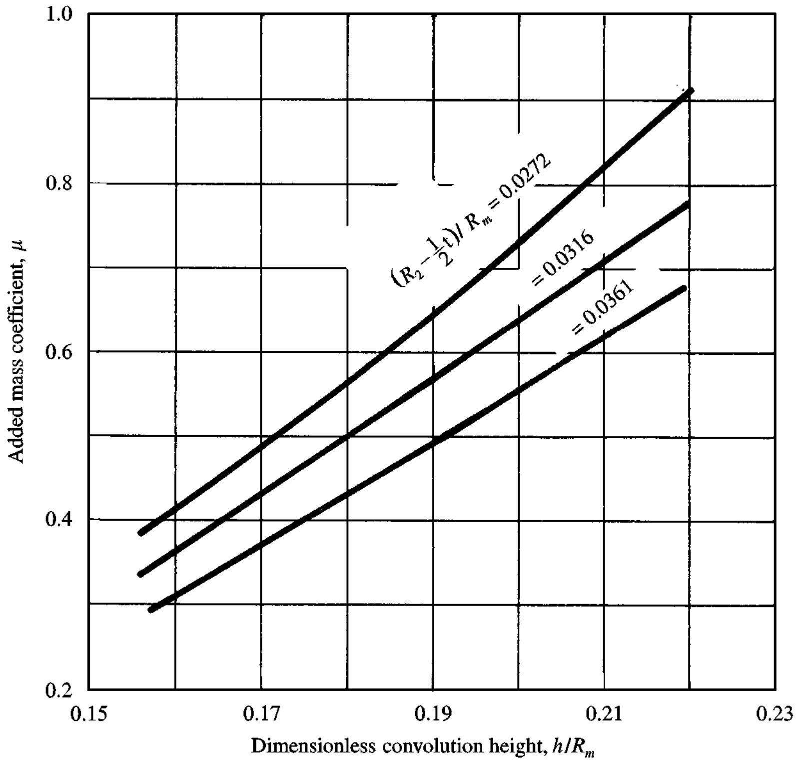

Figure 6. Half-convolution added mass coefficient, μ, for a range of typical bellows geometries.

Further refinement of the grid produced no change in the results. The program was written in Fortran which accommodated both the velocity potential, Φ, and the added mass, λ, calculation codes. The added mass calculation results for one half-convolution over a typical range of bellows geometries are presented graphically in Figure 6 in the form of the dimensionless coefficient of the added mass, μ, based on the mean radius of the bellows, R_m. Therefore, using the μ values taken from the graphs, the added mass λ can be calculated as

$$\lambda = \mu R_m^3 \rho_f, \tag{22}$$

or, using equation (14), the added mass caused by convolution distortion is

$$m_{f2} = \alpha_{f2k} \mu R_m^3 \rho_f. \tag{23}$$

The total added mass, using equations (4) and (23), can be expressed as

$$m_f = \left[\pi \left(R_m - \frac{h}{2} + \frac{2hR_2}{p} \right)^2 + \alpha_{f2k} \mu R_m^3 \right] \rho_f. \tag{24}$$

As was pointed out by a reviewer of this paper, the introduction of the half-convolution added mass, λ, is unnecessary. The same final result can be obtained without this intermediate step. However, the authors' computer code was developed to compute λ as defined in equation (18) and these results were used with equation (22) to generate the dimensionless added mass coefficient, μ, plotted in Figure 6. Therefore, the theoretical development including λ was presented here.

3. EXAMPLE CALCULATIONS AND DISCUSSION

In order to demonstrate the use of equation (24) for the total fluid-added mass of a bellows in transverse vibration and to evaluate the relative importance of the various components, calculations will be given for a particular case. The specific example was chosen because it was used in a subsequent experimental program. The physical parameters are: $R_m = 0{\cdot}0842$ m, $h = 0{\cdot}0157$ m, $R_1 = 0{\cdot}00353$ m, $R_2 = 0{\cdot}00248$ m, $t = 0{\cdot}368$ mm, the bellows live length, $l = 0{\cdot}156$ m, and the internal fluid density $\rho_f = 1000\,\mathrm{kg/m^3}$.

Assuming that the bellows is fixed against rotation at both ends, as would normally be the case for a single bellows expansion joint installed in a piping system, the mode function, X_k, and its second derivative can be found in standard texts on beam vibrations. Substitution of X_k and X_k'' into equation (15) and carrying out the integration yields

$$\alpha_{f2k} = 0{\cdot}666\,\frac{A_{1k}^2}{l^4}\left(R_m - \frac{h}{2}\right)^2 p, \tag{25}$$

where, for the first four modes, A_{1k} is 22·37, 61·67, 120·9, 199·9 for $k = 1, 2$, 3 and 4, respectively.

Using equations (24) and (25) together with Figure 6, the value of the mass per unit length of the three components m_b, m_{f1}, and m_{f2}, the total mass per unit length, m_{tot} and the ratio m_{f2}/m_{tot} are given in Table 1 for the first four modes of transverse vibration. It is seen that the convolution distortion component of added mass per unit length is only about 5.5% of the total in the first mode but that its significance increases substantially as the mode order increases. It is about 30% in the second mode and over 82% by the fourth mode.

The effect of convolution distortion on fluid added mass for bellows in transverse vibration has not been previously considered. To provide an indication of the importance of taking such a component into consideration, the natural frequency calculation procedure provided in the EJMA (1980) Standard was used to determine the first four transverse natural frequencies, with and without m_{f2} as given in Table 2. The percentage difference in the first mode frequency resulting from neglecting the fluid-added mass/unit length due to convolution distortion is less than 3% and, at least for this particular bellows, is acceptable from an engineering point of view. However, as the mode number increases, the convolution distortion component becomes increasingly important and cannot reasonably be ignored. It should be noted that the error caused by ignoring m_{f2} is unconservative, and actual natural frequency being lower than that predicted when $m_{f2} = 0$. It is important to note here that rotary inertia of the bellows cross-section is also ignored in the EJMA (1980) Standad. This makes these predictions for natural frequencies even more unconservative. Morishita *et al.*

TABLE 1
Components of bellows mass for various modes (kg/m)

Component of mass	Mode number 1	2	3	4
m_b		4·87		
m_{f1}		21·6		
m_{f2}	1·55	11·7	45·1	123
m_{tot}	28·0	38·2	71·6	149
m_{f2}/m_{tot}	0·055	0·307	0·630	0·826

TABLE 2
Comparison of frequency calculation results (Hz)

Mode number	EJMA Standard without m_{f2}	EJMA Standard with m_{f2}	% diff.
1	140	137	2·70
2	386	324	16·3
3	753	464	38·4
4	1252	535	57·3

(1986) included rotary inertia in their analysis, and its importance has been proven by Jakubauskas (1996). The predictive technique outlined here has been used in a much more comprehensive theoretical and experimental research program (Jakubauskas 1996). Experiments in both still and flowing fluids have validated the theory presented in this paper with errors in the transverse mode frequencies less than 5% in the first four modes (Jakubauskas 1996). The theoretical model for transverse bellows vibration, including the added mass components developed here, and comparison with experiments, are presented in Part II of this paper.

4. CONCLUSIONS

A theoretical model has been developed for determining the fluid-added mass of bellows expansion joints in transverse vibration. The total fluid added mass is treated in terms of rigid-body and convolution-shape distortion components. The results have been put in a form which enables relatively simple computation for a practical range of bellows. The results show that the component of added mass per unit length attributable to convolution distortion, which has been neglected in previous analyses, becomes increasingly important with increased mode number. For longer bellows, its effect may be small in the first mode but is not negligible in higher modes. For shorter bellows, the distortion component may be significant even in the first mode. Neglect of the distortion component of fluid added mass results in an underestimation of the total bellows mass per unit length and, therefore, an overestimation of the true transverse natural frequencies of bellows.

ACKNOWLEDGEMENTS

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APPENDIX: NOMENCLATURE

EI	beam bending stiffness
h	convolution height
k	vibration mode number
K	kinetic energy
K_i	kinetic energy of the ith convolution
l	the length of the bellows
m_b	bellows mass per unit length
m_{f1}	added fluid mass per unit length (rigid-body component)
m_{f2}	added fluid mass per unit length (distortion component)
m_f	total added fluid mass per unit length
M	local bending moment
N	number of convolutions
p	convolution pitch
R_m	mean radius of bellows
R_1	radius of convolution root
R_2	radius of convolution crown
S	half-convolution surface
t	nominal thickness of bellows material
$T(t)$	time harmonic function
U^*	normalized to unity relative convolution displacement
U	relative convolution displacement
U_x, U_y, U_z	convolution displacements in the coordinate directions
U_{xst}	axisymmetric static relative convolution displacement
w_k	transverse (bending) displacement of bellows axis in the kth mode
x, y, z	fixed coordinates
δ_i	half-convolution root relative displacement
ε	effective axial strain at convolution root
λ	half-convolution added fluid mass
μ	half-convolution added mass coefficient
ρ_f	density of fluid
Φ	velocity potential